

Mean:- $\bar{X} = \frac{\sum X}{N}$ ← For Raw data

$\bar{X} = A + \frac{\sum d}{N}$ ← For individual series
 $A \rightarrow$ Assumed value
 $d = (X-A)$

$\bar{X} = A + \frac{\sum fd}{N}$ ← For compound series
 $f \rightarrow$ Freq., $d = (X-A)$

Median:-
 Range data
 in ascending
 order

Med. = $\left(\frac{n+1}{2}\right)^{\text{th}}$ item ← For raw data

Med. = $L + \left[\frac{\frac{N}{2} - m}{f} \right] \times c$ ← For group compound series.

$Q_1 = \left(\frac{n+1}{4}\right)^{\text{th}}$ item, $Q_3 = 3\left(\frac{n+1}{4}\right)^{\text{th}}$ item

$Q.D = \frac{(Q_3 - Q_1)}{2}$, $\text{Coef. Q.D} = \frac{(Q_3 - Q_1)}{(Q_3 + Q_1)}$

Mode = $L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c$ $f_1 \rightarrow$ freq. of modal class.

Range = $L - S$, Co. of Range = $(L - S)/(L + S)$

Coef. of Mean. Dev. = $\frac{\text{Mean Dev.}}{\text{Mean or Med or Mode}} \times 100$

M.D. = $\frac{\sum |D|}{n}$ ← individual series $D = (X - \bar{X})$

= $\frac{\sum f|D|}{n}$ ← discrete series.

Coef. of
 Variation (C.V.)
 $= \frac{\sigma}{\bar{X}} \times 100$

S.D. = $\sigma = \sqrt{\frac{\sum (X - \bar{X})^2}{n}}$, for Assumed mean $\rightarrow \sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}$

I] Frequency:- Repeatables of particular value

Eg:- Scores:- 1, 1, 2, 2, 2, 2, 3, 3, 3, 4, 4, 5, 5, 5
 $\therefore$ Freq of 4 is 2, 5 is 3.

II] Freq. Distribution:-

1) Discrete / Ungrouped F.D.

Eg. Survey of 20 families, having no. of childrens per family

No. of child	T.M	Freq
0	1	1
1	2	2
2	2	2
3	0	5
4	5	5
5	1	1
6	1	3

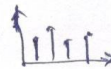
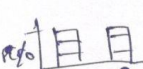
2) Continuous / Grouped F.D.

Wages $\rightarrow$ No. of Employees.

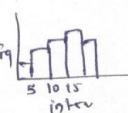
$\Rightarrow$ Class Limits, class intervals $\begin{cases} \text{Exclusive} \rightarrow 0-5, 5-10 \\ \text{Inclusive} \rightarrow 0-4, 5-9 \end{cases}$

open ended classes $\rightarrow$ Below 5 ----- 9 and above

III] Diagrams:-

Eg:- 1) Line Dig. , 2) Bar Dig. , 3) Multiple Bar 
 4) Percentage Bar Dig.  5) Pie Diagram

IV] Graphs.

1) Histogram 

2) Freq. Polygon 

V] Measures of Central Tendency.

1) Mean or Arithmetic Mean. $= \frac{\sum x}{N}$

2) Geometric Mean $= \sqrt[N]{x_1 \times x_2 \times \dots \times x_n}$

3) Harmonic Mean. $= N / \left[\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n} \right]$

Ⓐ Calculation of Mean - ~~Discrete~~ Individual series

1) Direct Method: $\bar{X} = \frac{\sum X}{N}$

2) Short Cut method $\bar{X} = A + \frac{\sum d}{N}$; $d = (X - A)$

Eg: - Monthly Consumption of milk by 5 families is.

105, 60, 80, 95, 120

Direct $\bar{X} = \frac{\sum X}{N} = \frac{460}{5} = 92$ Ltr.

Short cut
Let $A \rightarrow 80, N=5$
 $\therefore \bar{X} = 80 + \frac{\sum d}{N} = 80 + \frac{60}{5}$
 $= 80 + 12 = 92$ Ltr.

Sl. No.	Consumption	Ltr. $d = (X - A)$
1	105	25
2	60	-20
3	80	0
4	95	15
5	120	40
$N=5$	$\sum X=460$	$\sum d=60$

Ⓐ Means of Discrete / Ungrouped data

Direct: $\bar{X} = \frac{\sum fX}{\sum f}$

X	Wages	100	200	300	400	500	Σ
f	No. of workers	2	4	5	5	3	Σf=20
		200	800	1500	2000	1500	ΣfX=6300

$= \frac{6300}{20} = 315$ Rs.

Short cut
 $\bar{X} = A + \left(\frac{\sum f dx}{\sum f \text{ or } N} \right)$

$= 400 + \frac{-1700}{20}$
 $= 400 - 85$
 $= 315$

Let $A \rightarrow 400$

$dx = (X - A) = -300 | -200 | -100 | 0 | 100$

$\sum dx = -500$

$f dx = -600 | -800 | -600 | 0 | 300$

$\sum f dx = -1700$

Ex:- Given the freq. Dist. of first year students of particular college. Find Mean

'x'	Age (yrs)	15	16	17	18	19
'f'	No. of students	10	8	12	15	7
	$dx = (x - A) \rightarrow$	-1	0	1	2	3
'fx'		150	128	204	270	133

$\Sigma f = 52$
 $\Sigma fx = 885$

Direct:- $\bar{X} = \frac{\Sigma fx}{\Sigma f} = \frac{885}{52} = 17.02$

$f dx = -10, 0, 12, 30, 21$

Short Cut:- Let $A = 16$, $\therefore \Sigma dx = 5$ $\therefore \Sigma f dx = 53$

$\therefore \bar{X} = A + \left[\frac{\Sigma f dx}{\Sigma f} \right] = 16 + \left[\frac{53}{52} \right] = 16 + 1.02 = 17.02$

Ans = 17.02

Ex:- The following data shows distance covered by 100 persons to perform their routine jobs.

Distance Km	No. of persons (f)	Mid points (x)	fx	dx (x-A)	f dx
0-10	10	5	50	-10	-100
10-20	20	15	300	0	0
20-30	40	25	1000	10	400
30-40	30	35	1050	20	600
	$\Sigma f = 100$		$\Sigma fx = 2400$		$\Sigma f dx = 900$

Direct:- $\bar{X} = \frac{\Sigma fx}{\Sigma f} = \frac{2400}{100} = 24 \text{ Km}$

S. Cut:- $\bar{X} = A + \left[\frac{\Sigma f dx}{\Sigma f} \right] = 15 + \left(\frac{900}{100} \right) = 15 + 9 = 24 \text{ Km}$

Assume $A = 15$

x	Calculate mean:
0-10	5
10-20	15
20-30	25
30-40	35
40-50	45
50-60	55
60-70	65

★ Calculation of A.M. in unequal classes.
Calculate the Mean of the following series.

Weekly Consumption of Water (ltr)	No. of Families	Mid X	dx(X-A)	f dx
0-200	16	100	-600	-9600
200-500	20	350	-350	-7000
500-900	40	700	0	0
900-1500	24	1200	500	12000
1500-2000	30	1750	1050	31500
	$\Sigma f = 130$			$\Sigma f dx = 26900$

A=700

$$\begin{aligned}\bar{X} &= A + \left(\frac{\Sigma f dx}{\Sigma f} \right) = 700 + \left(\frac{26900}{130} \right) \\ &= 700 + 206.92 \\ &= \underline{\underline{906.92}}\end{aligned}$$

★ Find Combined Mean for the data given below.

$$n_1 = 20, \bar{x}_1 = 4, n_2 = 30, \bar{x}_2 = 3$$

$$\begin{aligned}\therefore \text{Combined Mean } \bar{X} &= \left[\frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} \right] = \frac{(20 \times 4) + (30 \times 3)}{20 + 30} \\ &= \frac{80 + 90}{50} = \frac{170}{50} = 3.4\end{aligned}$$

Median

$$Md = \left(\frac{n+1}{2} \right)^{\text{th}} \text{ item}$$

eg:- For odd no.s: 25, 18, 27, 10, 8, 30, 42, 20, 53

Ascending $\Rightarrow$ 8, 10, 18, 20, 25, 27, 30, 42, 53

$$Md = \left(\frac{9+1}{2} \right)^{\text{th}} \text{ item} = 5^{\text{th}} \text{ item} = 25$$

For Ex :- 5, 8, 12, 30, 18, 10, 2, 22

Ascending :- 2, 5, 8, 10, 12, 18, 22, 30

By formula: - $Md = \left(\frac{n+1}{2}\right)^{th} \text{ item} = \left(\frac{8+1}{2}\right)^{th} \text{ item} = 4.5^{th} \text{ item}$
 $\Rightarrow 4^{th} \text{ item} + \left(\frac{1}{2}\right)^{th} (5^{th} \text{ item} - 4^{th} \text{ item})$
 $\Rightarrow 10 + \left(\frac{1}{2}\right) [12 - 10] = 10 + 1 = \underline{\underline{11}}$

For continuous data:-

$$Md = L + \left[\frac{\frac{N}{2} - m}{f} \right] \times c$$

where :-
 L = Lower limit of Median class
 m = cumulative freq. of preceding median
 c = width of median class (interval)
 f = freq. of the Md. class
 N = total freq.

The following table gives the frequency distribution of 325 workers of a factory, according to their average monthly income in a certain year.

Income group (in Rs)	Number of workers
Below 100	1
100-150	20
150-200	42
200-250	55
250-300	62
300-350	45
350-400	30
400-450	25
450-500	15
500-550	18
550-600	10
600 and above	2
	325

Calculate median income

Solution:

Income group (Class-interval)	Number of workers (Frequency)	Cumulative frequency c.f
Below 100	1	1
100-150	20	21
150-200	42	63
200-250	55	118
250-300	62	180
300-350	45	225
350-400	30	255
400-450	25	280
450-500	15	295
500-550	18	313
550-600	10	323
600 and above	2	325
	325	

$$\frac{N}{2} = \frac{325}{2} = 162.5$$

$$= 300 - (0.39 \times 50)$$

$$300 - 19.5$$

$$= 280.5$$

Here $l = 250$, $N = 325$, $f = 62$, $c = 50$, $m = 118$

$$Md = 250 + \left(\frac{162.5 - 118}{62} \right) \times 50$$

$$= 250 + 35.89$$

$$= 285.89$$

$$200 + \left(\frac{162.5 - 180}{45} \right) \times 50$$

$$= 300 + \left(\frac{-17.5}{45} \right) \times 50$$

$$= 300 - \frac{875}{45}$$

Example 16:

Calculate median from the following data

Value	0-4	5-9	10-14	f	True class interval	c.f	25-29	30-34	35-39
Frequency	5	8	10	12	7	6		3	2
	0-4	5	0.5-4.5	5					
	5-9	8	4.5-9.5	13					
	10-14	10	9.5-14.5	23					
	15-19	12	14.5-19.5	35					
	20-24	7	19.5-24.5	42					
	25-29	6	24.5-29.5	48					
	30-34	3	29.5-34.5	51					
	35-39	2	34.5-39.5	53					
		53							

$$\left(\frac{N}{2} \right) = \left(\frac{53}{2} \right) = 26.5$$

$$Md = l + \frac{\frac{N}{2} - m}{f} \times c$$

$$= 14.5 + \frac{26.5 - 23}{12} \times 5$$

$$= 14.5 + 1.46 = 15.96$$

Example 17:

Following are the daily wages of workers in a textile. Find the median.

Wages (in Rs.)	Number of workers
less than 100	5
less than 200	12
less than 300	20
less than 400	32
less than 500	40
less than 600	45
less than 700	52
less than 800	60
less than 900	68
less than 1000	75

Solution :

We are given upper limit and less than cumulative frequencies. First find the class-intervals and the frequencies. Since the values are increasing by 100, hence the width of the class interval equal to 100.

Class interval	f	c.f
0-100	5	5
100-200	7	12
200-300	8	20
300- 400	12	32
400-500	8	40
500-600	5	45
600-700	7	52
700-800	8	60
800-900	8	68
900-1000	7	75
	75	

$$\left(\frac{N}{2}\right) = \left(\frac{75}{2}\right) = 37.5$$

$$Md = l + \left(\frac{\frac{N}{2} - m}{f} \right) \times c$$

$$= 400 + \left(\frac{37.5 - 32}{8} \right) \times 100 = 400 + 68.75 = 468.75$$

Example 18:

Find median for the data given below.

Marks	Number of students
Greater than 10	70
Greater than 20	62
Greater than 30	50
Greater than 40	38
Greater than 50	30
Greater than 60	24
Greater than 70	17
Greater than 80	9
Greater than 90	4

Solution :

Here we are given lower limit and more than cumulative frequencies.

Class interval	f	More than c.f	Less than c.f
10-20	8	70 $\nearrow 8$	8
20-30	12	62 $\nearrow 12$	20
30-40	12	50 $\nearrow 12$	32
40-50	8	38 $\nearrow 8$	40
50-60	6	30 $\nearrow 6$	46
60-70	7	24 $\nearrow 7$	53
70-80	8	17 $\nearrow 8$	61
80-90	5	9 $\nearrow 5$	66
90-100	4	4	70
	70		

$$\left(\frac{N}{2} \right) = \left(\frac{70}{2} \right) = 35$$

$$\begin{aligned}
 \text{Median} &= l + \left(\frac{\frac{N}{2} - m}{f} \right) \times c \\
 &= 40 + \left(\frac{35 - 32}{8} \right) \times 10 \\
 &= 40 + 3.75 \\
 &= 43.75
 \end{aligned}$$

Example 19:

Compute median for the following data.

Mid-Value	5	15	25	35	45	55	65	75
Frequency	7	10	15	17	8	4	6	7

Solution :

Here values in multiples of 10, so width of the class interval is 10.

Mid x	C.I	f	c.f
5	0-10	7	7
15	10-20	10	17
25	20-30	15	32
35	30-40	17	49
45	40-50	8	57
55	50-60	4	61
65	60-70	6	67
75	70-80	7	74
		74	

$$\left(\frac{N}{2} \right) = \left(\frac{74}{2} \right) = 37$$

$$\text{Median} = l + \left(\frac{\left(\frac{N}{2} \right) - m}{f} \right) \times c$$

$$= 30 + \left(\frac{37 - 32}{17} \right) \times 10$$

$$115$$

$$= 30 + 2.94 = 32.94$$

Quartiles :-

$$Q.D. = \frac{Q_3 - Q_1}{2}$$

$$\text{where } Q_1 = \left(\frac{n+1}{4}\right)^{\text{th}} \text{ item \& } Q_3 = 3\left(\frac{n+1}{4}\right)^{\text{th}} \text{ item}$$

Prob:- Compute quartiles for the given data.

25, 18, 30, 8, 15, 5, 10, 35, 40, 45

Sol:- Arrange in ascending order

5, 8, 10, 15, 18, 25, 30, 35, 40, 45

$$\begin{aligned} \therefore Q_1 &= \left(\frac{10+1}{4}\right)^{\text{th}} \text{ item} = (2.75)^{\text{th}} \text{ item} \\ &= 2^{\text{nd}} \text{ item} + \left(\frac{3}{4}\right)(3^{\text{rd}} \text{ item} - 2^{\text{nd}} \text{ item}) \\ &= 8 + \left(\frac{3}{4}\right)(10 - 8) = 8 + \frac{3 \times 2}{4} = 8 + 1.5 = 9.5 \end{aligned}$$

$$\begin{aligned} Q_3 &= 3\left(\frac{10+1}{4}\right)^{\text{th}} \text{ item} = 3(2.75)^{\text{th}} \text{ item} \\ &= (8.25)^{\text{th}} \text{ item} = 8^{\text{th}} \text{ item} + \left(\frac{1}{4}\right)(9^{\text{th}} - 8^{\text{th}}) \text{ item} \\ &= 35 + \frac{1}{4}(40 - 35) \\ &= 35 + \frac{5}{4} = 35 + 1.25 \\ &= \underline{\underline{36.25}} \end{aligned}$$

Prob 2 Compute quartiles for the discrete series

X	f	c.f
5	4	4
8	3	7
12	2	9
15	4	13
19	6	18
24	2	20
30	4	24

$$Q_1 = \left(\frac{N+1}{4} \right)^{th} \text{ item} = \left(\frac{24+1}{4} \right)^{th}$$

$$= \frac{25}{4} = (6.25)^{th} \text{ item} \approx 8$$

$$Q_3 = 18.75^{th} \text{ item} \Rightarrow Q_3 = 24$$

Prob 3 The following series relates to the marks secured by students in an examination. Find Quartiles.

Marks C.I	No. of Students f	c.f
0-10	11	11
10-20	18	(29) m_1
20-30	(25) f_1	54
30-40	28	82
40-50	30	112
50-60	33	(145) m_3
60-70	(22) f_3	167
70-80	15	182
80-90	12	194
90-100	10	(204)

$$\left(\frac{N}{4} \right) = 51, \quad 3 \left(\frac{N}{4} \right) = 153$$

$$Q_1 = L_1 + \frac{\frac{N}{4} - m_1}{f_1} \times C_1$$

$$= 20 + \left[\frac{51 - 29}{25} \right] \times 10$$

$$= 28.8$$

$$Q_3 = L_3 + \frac{3 \left(\frac{N}{4} \right) - m_3}{f_3} \times C_3$$

$$= 60 + \left[\frac{153 - 145}{22} \right] \times 10$$

$$= 60 + 3.64$$

$$= 63.64$$

Deciles :- 10 Equal Parts $D_1 \dots D_9$ i.e. 9 deciles

Ex:- Compute D_5 for the data given below

5, 24, 36, 12, 20, 8

Solⁿ:- 5, 8, 12, 20, 24, 36

$$D_5 = \left[\frac{5(n+1)}{10} \right]^{\text{th}} \text{ item} = \left[\frac{5(6+1)}{10} \right]^{\text{th}} \text{ item}$$

$$= 3.5^{\text{th}} \text{ item} = 3^{\text{rd}} \text{ item} + 0.5 [4^{\text{th}} - 3^{\text{rd}} \text{ item}]$$

$$= 12 + 0.5(20-12) = 12 + 4 = 16$$

★ Deciles for Grouped data:-

⇒ Calculate D_3 and D_7 for the given data

C.I	f	c.f
0-10	5	5
10-20	7	12
20-30	12	24
30-40	16	40
40-50	10	50
50-60	8	58
60-70	4	62

~~$D_3 = L + \frac{3(\frac{N}{10}) - m}{f} \times c$~~

$$D_3 \text{ item} = \left(\frac{3N}{10} \right)^{\text{th}} \text{ item}$$

$$= \left(\frac{3 \times 62}{10} \right)^{\text{th}} \text{ item}$$

$$= 18.6^{\text{th}} \text{ item}$$

i.e. interval lies betⁿ 20-30

$$\therefore D_3 = L + \left[\frac{3(\frac{N}{10}) - m}{f} \right] \times c$$

$$= 20 + \left(\frac{18.6 - 12}{12} \right) \times 10$$

$$= 20 + 5.5 = \underline{\underline{25.5}}$$

$$D_7 \text{ item} = \left(\frac{7N}{10} \right)^{\text{th}} \text{ item}$$

$$= \frac{7 \times 62}{10} = 43.4^{\text{th}} \text{ item}$$

i.e. interval class (40-50)

$$\therefore D_7 = L + \left[\frac{7(\frac{N}{10}) - m}{f} \right] \times c$$

$$= 40 + \left[\frac{43.4 - 40}{10} \right] \times 10$$

$$= 40 + 3.4 = \underline{\underline{43.4}}$$

★ Percentile :- 100 parts

Relationship : $P_{25} = Q_1$; $P_{50} = D_5 = Q_2 = \text{Median}$

$$P_{75} = Q_3$$

Ex) - Calculate P_{15} for the data given below :

5, 24, 36, 12, 20, 8.

$\Rightarrow$ 5, 8, 12, 20, 24, 36

$$\begin{aligned} P_{15} &= \left[\frac{15(n+1)}{100} \right]^{\text{th}} \text{ item} = \left(\frac{15 \times 7}{100} \right)^{\text{th}} \text{ item} = \left(\frac{105}{100} \right)^{\text{th}} \text{ item} \\ &= (1.05)^{\text{th}} \text{ item} = 1^{\text{st}} \text{ item} + 0.05(2^{\text{nd}} - 1^{\text{st}} \text{ item}) \\ &= 5 + 0.05(8 - 5) = 5 + 0.05(3) = 5 + 0.15 = \underline{\underline{5.15}} \end{aligned}$$

$\Rightarrow$ Ex) - Find P_{53} for following F.D.

C.I	f	cf
0-5	5	5
5-10	8	13
10-15	12	25
15-20	16	41
<u>20-25</u>	20	61
25-30	10	71
30-35	4	75
35-40	3	78

$$\begin{aligned} P_{53} &= L + \left[\frac{\frac{53(N+1)}{100} - m}{f} \right] \times c \\ &= 20 + \frac{\frac{53 \times 78}{100} - 41}{20} \times 5 \\ &= 20 + \left[\frac{\frac{4134}{100} - 41}{20} \right] \times 5 \\ &= 20 + \frac{41.34 - 41}{20} \times 5 \\ &= 20 + \left(\frac{0.34 \times 5}{20} \right) = 20 + 0.085 \\ &= \underline{\underline{20.085}} \end{aligned}$$

Mode:-

Eg:- 2, 7, 10, 15, 10, 17, 8, 10, 2

$$\therefore \text{Mode} = M_0 = 10$$

4. $\rightarrow$ 12, 10, 15, 24, 30 — (No mode)

4. $\rightarrow$ 7, 10, 15, 12, 7, 14, 24, 10, 7, 20, 10 — (Mode are 7 & 10)

(*) Continuous distribution :-

$$\text{Mode} = M_0 = l + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times c$$

$l \Rightarrow$ lower limit of modal class

$$\Delta_1 \Rightarrow (f_1 - f_0), \quad \Delta_2 \Rightarrow (f_1 - f_2)$$

$f_1 \rightarrow$ freq. of modal class

$f_0 \rightarrow$ freq. of class preceding modal class

$f_2 \rightarrow$ freq. of class after modal class

$$\text{Hence } M_0 = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c \right]$$

Ex:- Calculate Mode.

C.I	f
0-50	5
50-100	14
100-150	40
150-200	91
200-250	150
250-300	87
300-350	60
350-400	38
400 and above	15

Here highest freq. is 150. and corresponding

C.I 200-250.

$$\therefore l = 200, f_1 = 150, f_0 = 91, f_2 = 87$$

$$c = 50$$

$$\therefore M_0 = 200 + \left[\frac{150 - 91}{300 - 91 - 87} \times 50 \right]$$

$$= 200 + \left[\frac{59}{122} \times 50 \right] = 200 + \frac{2950}{122}$$

$$= 200 + 24.18 = \underline{\underline{224.18}}$$

W116 Find Mean, Med., Mode.

Age (In Year)	x	No. of Person	f	cf	cf	cf	cf	cf	cf
Under 5	2.5	0	0	-30	Under 40	37.5	22	106	5
Under 10	7.5	7	7	-25	45	42.5	29	135	10
15	12.5	10	17	-20	50	47.5	35	170	15
20	17.5	14	31	-15	55	52.5	41	211	20
25	22.5	16	47	-10	60	57.5	47	258	25
30	27.5	17	64	-5	61-65	62.5	50	308	30
30-35	32.5	20	84	0					

$\Sigma fdx = 4315$

$$\Rightarrow \text{Mean} = A + \frac{\Sigma fdx}{\Sigma f} = 32.5 + \frac{4315}{308} = 32.5 + 14.0 = \boxed{46.5}$$

$$\Rightarrow \text{Med} = L + \frac{\frac{N}{2} - m}{f} \times c; \text{ where } L = 30, \frac{N}{2} = 154, m = 64$$

$$f = 20, c = 5$$

$$= 30 + \left[\frac{154 - 64}{20} \times 5 \right] = 30 + \left(\frac{90}{20} \times 5 \right) = 30 + 22.5 = \boxed{52.5}$$

$$\Rightarrow \text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c; \text{ where } L = 60, f_1 = 50, f_0 = 47$$

$$f_2 = 0, c = 5$$

$$= 60 + \frac{50 - 47}{100 - 47 - 0} \times c \Rightarrow 60 + \frac{3}{53} \times 5$$

$$= 60 + 0.28 = \boxed{60.28}$$

By Karl Pearson,

$$\begin{aligned} \text{Mode} &= 3 \text{ median} - 2 \text{ mean} \\ &= (3 \times 52.5) - (2 \times 46.5) \\ &= 157.5 - 93 \\ &= \underline{\underline{64.5}} \end{aligned}$$

Q5/18 Calculate mean, median, mode for the following data pertaining to marks in statistics out of 140 marks for 80 students in a class.

Marks more than : 0 20 40 60 80 100 120

No. of Students : 80 76 50 28 18 9 3

Sol? - Convert more than to less than.

Class	More than	Diff.	Freq.	Cum. F.	Mid	dx	$f dx$
0-20	80	4	4	4	10	$(x-A)$ -60	-240
20-40	76	26	26	30	30	-40	-1040
40-60	50	22	22	52	50	-20	-440
60-80	28	10	10	62	70	0	0
80-100	18	9	9	71	90	20	180
100-120	9	6	6	77	110	40	240
120-140	3	3	3	80	130	60	180

$$\sum f dx = -1120$$

$$\therefore \bar{X} = A + \frac{\sum f dx}{\sum f} = 70 + \left[\frac{-1120}{80} \right] = 70 - 14$$

$$\text{Mean} = 56$$

$$\Rightarrow \text{Med} = L + \left[\frac{\frac{N}{2} - m}{f} \times C \right] \text{ where } L = 40, N = 80, m = 30, f = 22, C = 20.$$

$$= 40 + \left(\frac{40 - 30}{22} \times 20 \right) = 40 + \frac{200}{22} = 40 + 9.09 = 49.09$$

$$\Rightarrow \text{Mod} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times C \text{ where } L = 40, f_1 = 26, f_0 = 26, f_2 = 22, C = 20$$

$$= 40 + \left[\frac{26 - 26}{52 - 26 - 22} \times 20 \right] = 40 + \left[\frac{22}{26} \times 20 \right]$$

$$= 40 + (0.85 \times 20) = 40 + 17 = 57$$

Q17] The median and mode of the following wage distribution are known to be Rs. 3350 and Rs. 3400 resp. find the values of f_3, f_4, f_5 .

Wage in Rs.	No. of Emp.	C.f.
0 - 1000	4	4
1000 - 2000	16	20
2000 - 3000	f_3 m	$20 + f_3$
3000 - 4000	f_4 f	$20 + f_3 + f_4$
4000 - 5000	f_5	$20 + f_3 + f_4 + f_5$
5000 - 6000	6	$26 + f_3 + f_4 + f_5$
6000 - 7000	4	$30 + f_3 + f_4 + f_5$
Total	230	

$$N = 230$$

$$\therefore f_3 + f_4 + f_5 = 200 \quad \text{--- (i)}$$

$$\text{Med} = L + \left[\frac{\frac{N}{2} - m}{f} \right] \times c$$

$$\therefore 3350 = 3000 + \left[\frac{\frac{230}{2} - (20 + f_3)}{f_4} \right] \times 1000$$

$$\therefore 350 f_4 = \frac{(95 - f_3)}{f_4} \times 1000$$

$$\therefore 35 f_4 = 9500 - 100 f_3$$

$$\therefore 35 f_4 + 100 f_3 = 9500 \quad \text{--- (ii)}$$

$$\therefore 35 f_4 + 175 f_3 = 14000 \quad \text{--- (iv)}$$

$$\therefore -75 f_3 = -4500$$

$$\therefore f_3 = 60 \text{ put in (iv)}$$

$$\therefore f_4 + 300 = 400$$

$$\therefore f_4 = 100 \text{ put in (i)}$$

$$\therefore 60 + 100 + f_5 = 200 \Rightarrow f_5 = 40$$

$$\text{Mod} = L + \left[\frac{f_4 - f_3}{2f_4 - f_3 - f_5} \times c \right]$$

$$\therefore 3400 = 3000 + \left[\frac{f_4 - f_3}{2f_4 - f_3 - f_5} \times 1000 \right]$$

$$\therefore 400(2f_4 - f_3 - f_5) = 1000(f_4 - f_3)$$

$$4f_4 - 2f_3 - 2f_5 = 5f_4 - 5f_3$$

$$\therefore f_4 = 3f_3 - 2f_5 \quad \text{--- (iii)}$$

$$\therefore f_5 = f_4 - 2f_3$$

$$\therefore f_5 = \frac{3f_3 - f_4}{2} \text{ put in (i)}$$

$$\therefore f_3 + f_4 - 200 = \frac{f_4 - 3f_3}{2}$$

$$\therefore 2f_3 + 2f_4 - f_4 + 3f_3 = 400$$

$$\therefore f_4 + 5f_3 = 400 \quad \text{--- (v)}$$

$$\Rightarrow 35 f_4 + 175 f_3 = 14000 \quad \text{--- (iv)}$$

④ Empirical Relationship between Averages.

By Karl Pearson ; $\text{Mode} = 3 \text{ median} - 2 \text{ mean}$